

# 3N Scattering in a Three-Dimensional Operator Formulation

W. Glöckle<sup>1</sup>, I. Fachruddin<sup>2</sup>, Ch. Elster<sup>3</sup>, J. Golak<sup>4</sup>, R. Skibiński<sup>4</sup>, and H. Witała<sup>4</sup>

<sup>1</sup>*Institut für theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany*

<sup>2</sup>*Departemen Fisika Universitas Indonesia, Depok 16424, Indonesia*

<sup>3</sup>*Institute of Nuclear and Particle Physics,*

*Department of Physics and Astronomy,*

*Ohio University, Athens, OH 45701, USA and*

<sup>4</sup>*M. Smoluchowski Institute of Physics,*

*Jagiellonian University, PL-30059 Kraków, Poland*

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## Abstract

A recently developed formulation for a direct treatment of the equations for two- and three-nucleon bound states as set of coupled equations of scalar functions depending only on vector momenta is extended to three-nucleon scattering. Starting from the spin-momentum dependence occurring as scalar products in two- and three-nucleon forces together with other scalar functions, we present the Faddeev multiple scattering series in which order by order the spin-degrees can be treated analytically leading to 3D integrations over scalar functions depending on momentum vectors only. Such formulation is especially important in view of awaiting extension of 3N Faddeev calculations to projectile energies above the pion production threshold and applications of chiral perturbation theory 3N forces, which are to be most efficiently treated directly in such three-dimensional formulation without having to expand these forces into a partial wave basis.

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## I. INTRODUCTION

The three-nucleon (3N) system is the first nontrivial case to learn about the action of nucleon-nucleon (NN) and three-nucleon (3N) forces in bound states and scattering observables. Below about 200 MeV laboratory projectile energy 3N scattering can be well treated in a momentum space representation based on the Faddeev equations in a partial wave representation [1] to calculate elastic nucleon-deuteron (Nd) scattering as well as breakup processes. A different approach using a coordinate space representation and based on hyperspherical harmonics is equally precise for elastic Nd scattering [2].

Over many years a rich set of Nd data has been accumulated [1, 3, 4, 5], which is not only a very valuable source of information about the spin- and momentum-dependence of nuclear forces but also about the reaction mechanism of multiple rescattering processes. The nuclear forces under consideration today are on the one hand the so-called (semi)-phenomenological high precision forces [6, 7, 8], describing the NN data up to the pion production threshold perfectly well. The three-nucleon forces on a semi-phenomenological level are much less developed [9] and are not constructed in a consistent manner with respect to corresponding NN forces. Nevertheless those NN and 3N forces describe 3N data (elastic as well as breakup cross sections and numerous spin observables) often spectacularly well. However, there are exceptions where one finds serious discrepancies between that theoretical prediction and the data, especially for some spin observables at the higher region of that energy range [3, 4, 5].

In recent years effective chiral perturbation theory links nuclear forces to the symmetry of QCD and diagrammatically builds up the nuclear forces in a systematic expansion [10]. In this fashion NN, 3N and even 4N forces are at present consistently generated up to next-to-next-to-next leading order ( $N^3\text{LO}$ ). Applications of these forces in the few-nucleon sector [11] and for light nuclei [12] are quite successful. However, the effective theory is limited in the energy regime it can be applied to, which is related to the smallness of the parameter underlying that expansion. A typical upper limit for the applicability of those chiral forces is 100-150 MeV nucleon laboratory projectile energy.

Considering the nucleon projectile energies above the pion production threshold and even going into the GeV region, no similar systematic approach to nuclear forces is yet available. However, this energy region imposes challenging questions not only about the nuclear force, but also about the underlying reaction mechanism. One should expect an increased importance of 3N forces, the influence of baryon resonances, meson production, to name a few. Another challenging question is an investigation of the limit, where hadronic degrees of freedom apply and where subnuclear

degrees of freedom must be explicitly considered. In order to enter this energy region in the 3N sector various challenges have to be overcome. On the technical side the standard partial-wave decomposition (PWD) has to be given up due to the strongly increasing number of partial wave states that need to be summed for a converged result. To face this challenge the direct use of momentum vectors, i.e. a 3-dimensional (3D) formulation of the problem turned out to be a promising path. This has been documented in various studies for three-boson scattering carried out in a Faddeev scheme in momentum space [13]. In addition, the high energy region requires that Galilean invariance has to be replaced by Poincaré invariance. For the case of three-boson scattering this has successfully been achieved [14, 15], where the Poincaré invariant Faddeev equations have been solved for projectile energies in the GeV regime. As already said the enormous challenge will be to develop the underlying dynamics.

It turns out that Poincaré invariant formulation of the 3N system is already important at quite low laboratory energy. Relativistic effects are discernable in some regions of the breakup phase-space starting at the energy of the incoming nucleon about 65 MeV [16]. For Nd elastic scattering vector analyzing power they contribute to the famous analyzing power puzzle at energies around 10 MeV [17].

In this paper we shall focus on the question how to incorporate spin and isospin degrees of freedom into the bosonic three-body calculations so that the successful approach of Ref. [13] is applicable to nucleons. The aim is to reduce the formulation with spin/isospin degrees of freedom to scalar, spin independent functions of vector momenta in the same spirit as already presented for the 2N and 3N bound states in Ref. [18]. The idea is to use the original structure of the NN forces consisting of scalar operators in spin- and momentum-space and scalar functions which only depend on momenta. This carries over to the NN t-operator which is a central building block in the Faddeev scheme. In addition, 3N forces appear naturally in this formulation.

One form of the Faddeev equations for 3N scattering is based on the multiple scattering series for the breakup process which can be summed into a Faddeev integral equation for a transition operator  $T|\Phi\rangle$  in our standard notation [19]. Here  $|\Phi\rangle$  is the initial product state of a deuteron and a momentum eigenstate of the projectile nucleon. Analogous to the NN t-operator there will be an operator form for three-nucleon transition operator  $T|\Phi\rangle$ . However the number of scalar spin-momentum operators will be enormously high, which makes it not advisable to rewrite the Faddeev equation for  $T|\Phi\rangle$  into a coupled set of equations for the accompanying scalar momentum dependent functions. The experience with high energy three-boson scattering [13] suggests that it is promising to generate instead the multiple scattering series, which will automatically generate

the operator expansion order by order. We want to use this insight as starting point for our study.

In Section II we introduce the necessary formal ingredients. The lowest order in the multiple scattering series is worked out in Section III. The next term, which is second order in the NN t-operator, is constructed in Section IV. In this order, the free 3N propagator appears for the first time, leading to the notorious moving logarithmic singularities. This can be avoided totally as shown in [20, 21]. We apply this new method in which the logarithmic singularities are replaced by single poles, which then can be handled in a similar fashion as the poles in the 2N Lippmann-Schwinger equation. Having the second order under control it is obvious to go to the next order. However, we will not give this obvious continuation explicitly. The inclusion of 3NF's, which is most interesting from a physical point of view will be skipped in this first formal attempt. However, we do not expect principal difficulty according to the experience for the 3N bound state, where the inclusion of 3NF's has been worked out explicitly [18]. The calculation of observables based on the 3-dimensional form is described in Section V. Various Appendices provide further information. Finally we conclude in Section VI.

## II. THE FORMAL INGREDIENTS

Our standard form of the Faddeev equation for 3N scattering is given by [1, 19]

$$\begin{aligned} T|\phi\rangle &= tP|\phi\rangle + (1 + tG_0)V_4^{(1)}(1 + P)|\phi\rangle + tPG_0T|\phi\rangle \\ &+ (1 + tG_0)V_4^{(1)}(1 + P)T|\phi\rangle, \end{aligned} \quad (1)$$

where  $t$  is the t-operator for the NN pair 23,  $G_0$  the free 3N propagator,  $P$  the sum of a cyclical and an anticyclical permutation and  $V_4^{(1)}$  the part of the 3NF which is symmetrical under exchange of nucleons 2 and 3. Here we arbitrarily choose nucleon 1 as being the spectator.

Knowing  $T|\phi\rangle$  amplitudes for Nd elastic and breakup scattering are given by the matrix elements [1, 22]

$$\langle\Phi'|U|\Phi\rangle = \langle\Phi'|PG_0^{-1} + PT|\Phi\rangle, \quad (2)$$

$$\langle\Phi_0|U_0|\Phi\rangle = \langle\Phi_0|(1 + P)T|\Phi\rangle. \quad (3)$$

The state describing three free nucleons is given by  $|\Phi_0\rangle$ .

The iteration of Eq. (1) generates the Faddeev multiple scattering series. When neglecting 3NFs, the iteration leads to

$$T|\Phi\rangle = tP|\Phi\rangle + tPG_0tP|\Phi\rangle + \dots \quad (4)$$

As we learned from 3-boson scattering driven by a 2-body force of Malfliet-Tjon type [13] which incorporates typical properties of the NN force, namely an intermediate range attraction and a short range repulsion, the convergence of the series from Eq. (4) improves with increasing energy [23].

First, we introduce the three possible 3N isospin states  $|\gamma_a\rangle = |(t_a \frac{1}{2})T_a M_T\rangle$ :

$$\begin{aligned} |\gamma_0\rangle &= |(0\frac{1}{2})\frac{1}{2}M_T\rangle, \\ |\gamma_1\rangle &= |(1\frac{1}{2})\frac{1}{2}M_T\rangle, \\ |\gamma_2\rangle &= |(1\frac{1}{2})\frac{3}{2}M_T\rangle, \end{aligned} \quad (5)$$

in which the 2N isospin  $t$  is coupled with the isospin  $\frac{1}{2}$  of the third particle to the total isospin  $T = \frac{1}{2}$  or  $\frac{3}{2}$ . As is well known [24] in isospin space the 2N t-operator has the form

$$t = \sum_{ab} |\gamma_a\rangle t_{ab} \langle \gamma_b|. \quad (6)$$

We assume conservation of  $t_a$  but allow for charge independence and charge symmetry breaking which leads to the coupling of  $T = \frac{1}{2}$  and  $\frac{3}{2}$  states:

$$t_{ab} = \delta_{t_a t_b} t_{t_a T_a T_b}. \quad (7)$$

The linear combination of  $np$  t-operators in  $t = 0$  and  $t = 1$  states with  $pp$  ( $nn$ ) t-operators for proton-deuteron ( $pd$ ) ( $neutron-deuteron$  ( $nd$ )) scattering is given in [25]. Furthermore, the permutation operator  $P$  in the 3N isospin space reads [24]

$$\langle \gamma_a | P | \gamma_b \rangle = \delta_{T_a T_b} F_{t_a T_a T_b} (P_{12}^{sm} P_{23}^{sm} + (-)^{t_a + t_b} P_{13}^{sm} P_{23}^{sm}), \quad (8)$$

where  $F_{t_a T_a T_b}$  is essentially a  $6j$ -symbol [24] and  $P_{ij}^{sm}$  are transpositions of the nucleons  $ij$  acting only in spin and momentum spaces. The 3N momentum space is spanned by the standard Jacobi momenta  $\vec{p}$  and  $\vec{q}$  [22]. Combining spin/isospin space with the momentum space leads to the permutation operator

$$\begin{aligned} \langle \vec{p} \vec{q} | P_{12}^{sm} P_{23}^{sm} + (-)^{t_a + t_b} P_{13}^{sm} P_{23}^{sm} | \vec{p}' \vec{q}' \rangle = \\ \delta(\vec{p} - \vec{\pi}(\vec{q} \vec{q}')) \delta(\vec{p}' - \vec{\pi}'(\vec{q} \vec{q}')) P_{12}^s P_{23}^s + (-)^{t_a + t_b} \delta(\vec{p} + \vec{\pi}(\vec{q} \vec{q}')) \delta(\vec{p}' + \vec{\pi}'(\vec{q} \vec{q}')) P_{13}^s P_{23}^s \end{aligned} \quad (9)$$

with

$$\begin{aligned} \vec{\pi}(\vec{q}, \vec{q}') &= \frac{1}{2} \vec{q} + \vec{q}', \\ \vec{\pi}'(\vec{q}, \vec{q}') &= -\vec{q} - \frac{1}{2} \vec{q}'. \end{aligned} \quad (10)$$

Next we use the operator form expansion of the off-shell NN  $t$ -operator [18, 26] in the 3N spin and momentum spaces

$$\langle \vec{p}\vec{q} | t_{t_a T_a T_b} | \vec{p}' \vec{q}' \rangle = \sum_j t_{t_a T_a T_b}^{(j)}(\vec{p}, \vec{p}', E_q) w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{p}') \delta(\vec{q} - \vec{q}') , \quad (11)$$

where  $E_q = E - \frac{3}{4m}q^2$  and  $E$  the total c.m. energy. Due to parity and time reversal invariance exactly 6 terms of scalar spin-momentum operators  $w_j$  are possible [27]. They are

$$\begin{aligned} w_1(\vec{\sigma}(1), \vec{\sigma}(2), \vec{p}, \vec{p}') &= 1 \\ w_2(\vec{\sigma}(1), \vec{\sigma}(2), \vec{p}, \vec{p}') &= \vec{\sigma}(1) \cdot \vec{\sigma}(2) \\ w_3(\vec{\sigma}(1), \vec{\sigma}(2), \vec{p}, \vec{p}') &= (\vec{\sigma}(1) + \vec{\sigma}(2)) \cdot (\vec{p} \times \vec{p}') \\ w_4(\vec{\sigma}(1), \vec{\sigma}(2), \vec{p}, \vec{p}') &= \vec{\sigma}(1) \cdot (\vec{p} \times \vec{p}') \vec{\sigma}(2) \cdot (\vec{p} \times \vec{p}') \\ w_5(\vec{\sigma}(1), \vec{\sigma}(2), \vec{p}, \vec{p}') &= \vec{\sigma}(1) \cdot (\vec{p} + \vec{p}') \vec{\sigma}(2) \cdot (\vec{p} + \vec{p}') \\ w_6(\vec{\sigma}(1), \vec{\sigma}(2), \vec{p}, \vec{p}') &= \vec{\sigma}(1) \cdot (\vec{p} - \vec{p}') \vec{\sigma}(2) \cdot (\vec{p} - \vec{p}'). \end{aligned} \quad (12)$$

Using this operator representation of the NN force, the functions  $t_{t_a T_a T_b}^{(j)}(\vec{p}, \vec{p}', E_q)$  of Eq. (11) are scalar functions and depend only on three variables, the magnitudes of the vectors  $\vec{p}$  and  $\vec{p}'$ , and the angle between them,  $\hat{p} \cdot \hat{p}'$ .

Finally, we use the operator form of the initial state as given in Ref. [28]

$$\begin{aligned} \langle \vec{p}\vec{q} | \langle \gamma_0 | \Phi \rangle &\equiv \langle \vec{p}\vec{q} | \phi \rangle \\ &= (\phi_1(p) + \phi_2(p)(\vec{\sigma}(2) \cdot \vec{p}\vec{\sigma}(3) \cdot \vec{p} - \frac{1}{3}p^2) | 1m_d \rangle | m_{10} \rangle \delta(\vec{q} - \vec{q}_0) \\ &\equiv \sum_{k=1}^2 \phi_k(p) O_k(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}) | 1m_d \rangle | m_{10} \rangle \delta(\vec{q} - \vec{q}_0) . \end{aligned} \quad (13)$$

Here  $\phi_1(p)$  and  $\phi_2(p)$  are proportional to the standard s- and d-wave components of the deuteron wave function. The state  $|1m_d\rangle$  is the pure spin 1 two-nucleon state with spin magnetic quantum number  $m_d$ , and  $m_{10}$  is the initial state spin magnetic quantum number of the projectile nucleon. The vector  $\vec{q}_0$  is the initial relative momentum in the Nd system. All these are the necessary ingredients to work out the terms in the multiple scattering series of Eq. (4).

### III. THE FIRST ORDER IN $t$

We consider the first order term in Eq. (4) and use Eqs. (6) - (9), (11) and (13) to obtain

$$\langle \vec{p}\vec{q} | \langle \gamma_a | tP | \Phi \rangle = F_{t_a t_0 T_0} \langle \vec{p}\vec{q} | t_{t_a T_a T_0} (P_{12}^{sm} P_{23}^{sm} + (-)^{t_a+t_0} P_{13}^{sm} P_{23}^{sm}) | \phi \rangle . \quad (14)$$

Here we used the isospin property of the initial state:  $t_0 = 0, T_0 = 1/2$ . Then

$$\begin{aligned}
& \langle \vec{p}\vec{q} | \langle \gamma_a | tP | \Phi \rangle \\
&= F_{t_a t_0 T_0} \int d^3 p' d^3 q' \sum_j t_{t_a T_a T_0}^{(j)}(\vec{p}, \vec{p}', E_q) w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{p}') \delta(\vec{q} - \vec{q}') \\
&\quad \int d^3 p'' d^3 q'' (\delta(\vec{p}' - \vec{\pi}(\vec{q}' \vec{q}'')) \delta(\vec{p}'' - \vec{\pi}'(\vec{q}' \vec{q}'')) P_{12}^s P_{23}^s \\
&+ (-)^{t_a + t_0} \delta(\vec{p}' + \vec{\pi}(\vec{q}' \vec{q}'')) \delta(\vec{p}'' + \vec{\pi}'(\vec{q}' \vec{q}'')) P_{13}^s P_{23}^s) \\
&\quad \sum_{k=1}^2 \phi_k(p'') O_k(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}'') |1m_d\rangle |m_{10}\rangle \delta(\vec{q}'' - \vec{q}_0) \\
&= F_{t_a t_0 T_0} \left[ \sum_j t_{t_a T_a T_0}^{(j)}(\vec{p}, \vec{\pi}(\vec{q}\vec{q}_0), E_q) w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}(\vec{q}\vec{q}_0)) P_{12}^s P_{23}^s \right. \\
&\quad \sum_{k=1}^2 \phi_k(|\vec{\pi}'(\vec{q}\vec{q}_0)|) O_k(\vec{\sigma}(2), \vec{\sigma}(3), \vec{\pi}'(\vec{q}\vec{q}_0)) |1m_d\rangle |m_{10}\rangle \\
&+ (-)^{t_a + t_0} \sum_j t_{t_a T_a T_0}^{(j)}(\vec{p}, -\vec{\pi}(\vec{q}\vec{q}_0), E_q) w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, -\vec{\pi}(\vec{q}\vec{q}_0)) P_{13}^s P_{23}^s \\
&\quad \left. \sum_{k=1}^2 \phi_k(|\vec{\pi}'(\vec{q}\vec{q}_0)|) O_k(\vec{\sigma}(2), \vec{\sigma}(3), \vec{\pi}'(\vec{q}\vec{q}_0)) |1m_d\rangle |m_{10}\rangle \right] . \tag{15}
\end{aligned}$$

Note that we used the fact that the two operators  $O_k$  from Eq. (13) depend quadratically on the momenta. Then we define

$$\begin{aligned}
& w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}) P_{12}^s P_{23}^s O_k(\vec{\sigma}(2), \vec{\sigma}(3), \vec{\pi}') \\
&= w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}) O_k(\vec{\sigma}(3), \vec{\sigma}(1), \vec{\pi}') P_{12}^s P_{23}^s \\
&\equiv a_{jk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}, \vec{\pi}') P_{12}^s P_{23}^s , \tag{16}
\end{aligned}$$

and

$$\begin{aligned}
& w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, -\vec{\pi}) P_{13}^s P_{23}^s O_k(\vec{\sigma}(2), \vec{\sigma}(3), \vec{\pi}') \\
&= w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, -\vec{\pi}) O_k(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\pi}') P_{13}^s P_{23}^s \\
&\equiv b_{jk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}, \vec{\pi}') P_{13}^s P_{23}^s . \tag{17}
\end{aligned}$$

The scalar expressions  $a_{jk}$  and  $b_{jk}$  have to be worked out such that each  $\vec{\sigma}(i)$  occurs only once.

The explicit expressions are given in Appendix A.

Inserting then Eqs. (16) and (17) into Eq. (15) yields

$$\begin{aligned}
& \langle \vec{p}\vec{q} | \langle \gamma_a | tP | \Phi \rangle \\
&= \sum_{jk} t_{t_a T_a T_0}^{(j)}(\vec{p}, \vec{\pi}, E_q) \phi_k(|\vec{\pi}'|) F_{t_a t_0 T_0} \\
&\quad a_{jk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}, \vec{\pi}') P_{12}^s P_{23}^s |1m_d\rangle |m_{10}\rangle \\
&+ \sum_{jk} t_{t_a T_a T_0}^{(j)}(\vec{p}, -\vec{\pi}, E_q) \phi_k(|\vec{\pi}'|) F_{t_a t_0 T_0} (-)^{t_a + t_0}
\end{aligned}$$

$$b_{jk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}, \vec{\pi}') P_{13}^s P_{23}^s |1m_d\rangle |m_{10}\rangle) . \quad (18)$$

We see the expected structure, a sum over the product of scalar operators ( $a_{jk}, b_{jk}$ ) multiplied by scalar functions. Note that

$$\begin{aligned} a_{jk} &= a_{jk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}(\vec{q}\vec{q}_0), \vec{\pi}'(\vec{q}\vec{q}_0)) , \\ b_{jk} &= b_{jk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}(\vec{q}\vec{q}_0), \vec{\pi}'(\vec{q}\vec{q}_0)) . \end{aligned} \quad (19)$$

It remains to display the singularity structure for the calculation of the physical amplitudes and the treatment of the second order.

The NN t-matrix in the pd isospin space has the general structure

$$t_{t_a T_a T_b} = \delta_{t_a 0} t_{0\frac{1}{2}\frac{1}{2}} + \delta_{t_a 1} t_{1T_a T_b} , \quad (20)$$

where

$$t_{0\frac{1}{2}\frac{1}{2}} = t_{np}^{00} , \quad (21)$$

$$t_{1T_a T_b} = \alpha_{T_a T_b} t_{np}^{10} + \beta_{T_a T_b} t_{pp}^{1-1} , \quad (22)$$

and  $\alpha_{T_a T_b}, \beta_{T_a T_b}$  are numerical values related to Clebsch Gordon coefficients.

The np t-matrix  $t_{np}^{00}$  for  $t = 0$  has a deuteron pole

$$t_{np}^{00}(\vec{p}, \vec{p}', E_q) \equiv \frac{\hat{t}_{np}^{00}(\vec{p}, \vec{p}', E_q)}{E_q + i\epsilon - E_d} . \quad (23)$$

Therefore, Eq. (18) can be decomposed further,

$$\begin{aligned} &\langle \vec{p}\vec{q} | \langle \gamma_a | tP | \Phi \rangle \\ &= \sum_{jk} (\delta_{t_a 0} \frac{\hat{t}_{np}^{00,(j)}(\vec{p}, \vec{\pi}, E_q)}{E_q + i\epsilon - E_d} + \delta_{t_a 1} t_{1T_a T_0}^{(j)}(\vec{p}, \vec{\pi}, E_q)) \phi_k(|\vec{\pi}'|) F_{t_a t_0 T_0} \\ &\quad a_{jk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}, \vec{\pi}') P_{12}^s P_{23}^s |1m_d\rangle |m_{10}\rangle \\ &+ \sum_{jk} (\delta_{t_a 0} \frac{\hat{t}_{np}^{00,(j)}(\vec{p}, -\vec{\pi}, E_q)}{E_q + i\epsilon - E_d} + \delta_{t_a 1} t_{1T_a T_0}^{(j)}(\vec{p}, -\vec{\pi}, E_q)) \phi_k(|\vec{\pi}'|) F_{t_a t_0 T_0}(-)^{t_a + t_0} \\ &\quad b_{jk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}, \vec{\pi}') P_{13}^s P_{23}^s |1m_d\rangle |m_{10}\rangle) , \end{aligned} \quad (24)$$

which retains the structure of scalar spin-momentum dependent operators and momentum dependent scalar functions.

#### IV. THE SECOND ORDER IN $t$

Next we consider the second order term in Eq. (4) and perform the isospin projections according to Refs. (6) and (8)

$$\langle \gamma_a | tP G_0 tP | \Phi \rangle = t_{ab} \langle \gamma_b | P | \gamma_c \rangle G_0 \langle \gamma_c | tP | \Phi \rangle$$



$$\begin{aligned}
&= \delta_{t_a t_b} t_{t_a T_a T_b} \delta_{T_b T_c} F_{t_b t_c T_b} (P_{12}^{sm} P_{23}^{sm} + (-)^{t_b+t_c} P_{13}^{sm} P_{23}^{sm}) G_0 \langle \gamma_c | tP | \Phi \rangle \\
&= F_{t_a t_c T_b} t_{t_a T_a T_b} (P_{12}^{sm} P_{23}^{sm} + (-)^{t_a+t_c} P_{13}^{sm} P_{23}^{sm}) G_0 \delta_{T_b T_c} \langle \gamma_c | tP | \Phi \rangle .
\end{aligned} \tag{25}$$

Then we insert Eqs. (11) and (9)

$$\begin{aligned}
&\langle \vec{p} \vec{q} | \langle \gamma_a | tP G_0 tP | \Phi \rangle \\
&= F_{t_a t_c T_b} \int d^3 p' \sum_j t_{t_a T_a T_b}^{(j)}(\vec{p}, \vec{p}', E_q) w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{p}') \\
&\quad \int d^3 p_2 d^3 q_2 \langle \vec{p}' \vec{q} | (P_{12}^{sm} P_{23}^{sm} + (-)^{t_a+t_c} P_{13}^{sm} P_{23}^{sm}) | \vec{p}_2 \vec{q}_2 \rangle G_0(\vec{p}_2 \vec{q}_2) \\
&\quad \delta_{T_b T_c} \langle \vec{p}_2 \vec{q}_2 | \langle \gamma_c | tP | \Phi \rangle \\
&= F_{t_a t_c T_b} \int d^3 p' \sum_j t_{t_a T_a T_b}^{(j)}(\vec{p}, \vec{p}', E_q) w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{p}') \\
&\quad \int d^3 p_2 d^3 q_2 (\delta(\vec{p}' - \vec{\pi}(\vec{q} \vec{q}_2)) \delta(\vec{p}_2 - \vec{\pi}'(\vec{q} \vec{q}_2)) P_{12}^s P_{23}^s \\
&\quad + (-)^{t_a+t_c} \delta(\vec{p}' + \vec{\pi}(\vec{q} \vec{q}_2)) \delta(\vec{p}_2 + \vec{\pi}'(\vec{q} \vec{q}_2)) P_{13}^s P_{23}^s) \\
&\quad G_0(\vec{p}_2 \vec{q}_2) \delta_{T_b T_c} \langle \vec{p}_2 \vec{q}_2 | \langle \gamma_c | tP | \Phi \rangle \\
&= F_{t_a t_c T_b} \sum_j \int d^3 q_2 \left[ t_{t_a T_a T_b}^{(j)}(\vec{p}, \vec{\pi}(\vec{q} \vec{q}_2), E_q) w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}(\vec{q} \vec{q}_2)) \right. \\
&\quad P_{12}^s P_{23}^s G_0(\vec{\pi}'(\vec{q} \vec{q}_2) \vec{q}_2) \delta_{T_b T_c} \langle \vec{\pi}'(\vec{q} \vec{q}_2) \vec{q}_2 | \langle \gamma_c | tP | \Phi \rangle \\
&\quad + (-)^{t_a+t_c} t_{t_a T_a T_b}^{(j)}(\vec{p}, -\vec{\pi}(\vec{q} \vec{q}_2), E_q) w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, -\vec{\pi}(\vec{q} \vec{q}_2)) \\
&\quad \left. P_{13}^s P_{23}^s G_0(\vec{\pi}'(\vec{q} \vec{q}_2) \vec{q}_2) \delta_{T_b T_c} \langle -\vec{\pi}'(\vec{q} \vec{q}_2) \vec{q}_2 | \langle \gamma_c | tP | \Phi \rangle \right] .
\end{aligned} \tag{26}$$

Finally we use the expression from Eq. (18) for the first order term

$$\begin{aligned}
&\langle \vec{p} \vec{q} | \langle \gamma_a | tP G_0 tP | \Phi \rangle \\
&= F_{t_a t_c T_b} \sum_j \int d^3 q_2 \left[ t_{t_a T_a T_b}^{(j)}(\vec{p}, \vec{\pi}(\vec{q} \vec{q}_2), E_q) w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}(\vec{q} \vec{q}_2)) \right. \\
&\quad P_{12}^s P_{23}^s G_0(\vec{\pi}'(\vec{q} \vec{q}_2) \vec{q}_2) \delta_{T_b T_c} \\
&\quad \sum_{lk} (t_{t_c T_b T_0}^{(l)}(\vec{\pi}'(\vec{q} \vec{q}_2), \vec{\pi}(\vec{q}_2, \vec{q}_0), E_{q_2}) \phi_k(|\vec{\pi}'(\vec{q}_2, \vec{q}_0)|) F_{t_c t_0 T_0} \\
&\quad a_{lk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{\pi}'(\vec{q} \vec{q}_2), \vec{\pi}(\vec{q}_2, \vec{q}_0), \vec{\pi}'(\vec{q}_2, \vec{q}_0)) P_{12}^s P_{23}^s |1m_d\rangle |m_{10}\rangle \\
&\quad + t_{t_c T_b T_0}^{(l)}(\vec{\pi}'(\vec{q} \vec{q}_2), -\vec{\pi}(\vec{q}_2, \vec{q}_0), E_{q_2}) \phi_k(|\vec{\pi}'(\vec{q}_2, \vec{q}_0)|) F_{t_c t_0 T_0} (-)^{t_c+t_0} \\
&\quad b_{lk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{\pi}'(\vec{q} \vec{q}_2), \vec{\pi}(\vec{q}_2 \vec{q}_0), \vec{\pi}'(\vec{q}_2 \vec{q}_0)) P_{13}^s P_{23}^s |1m_d\rangle |m_{10}\rangle) \\
&\quad + (-)^{t_a+t_c} t_{t_a T_a T_b}^{(j)}(\vec{p}, -\vec{\pi}(\vec{q} \vec{q}_2), E_q) w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, -\vec{\pi}(\vec{q} \vec{q}_2)) \\
&\quad P_{13}^s P_{23}^s G_0(\vec{\pi}'(\vec{q} \vec{q}_2) \vec{q}_2) \delta_{T_b T_c} \\
&\quad \sum_{lk} (t_{t_c T_b T_0}^{(l)}(-\vec{\pi}'(\vec{q} \vec{q}_2), \vec{\pi}(\vec{q}_2, \vec{q}_0), E_{q_2}) \phi_k(|\vec{\pi}'(\vec{q}_2, \vec{q}_0)|) F_{t_c t_0 T_0} \\
&\quad a_{lk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), -\vec{\pi}'(\vec{q} \vec{q}_2), \vec{\pi}(\vec{q}_2, \vec{q}_0), \vec{\pi}'(\vec{q}_2, \vec{q}_0)) P_{12}^s P_{23}^s |1m_d\rangle |m_{10}\rangle \\
&\quad + t_{t_c T_b T_0}^{(l)}(-\vec{\pi}'(\vec{q} \vec{q}_2), -\vec{\pi}(\vec{q}_2, \vec{q}_0), E_{q_2}) \phi_k(|\vec{\pi}'(\vec{q}_2, \vec{q}_0)|) F_{t_c t_0 T_0} (-)^{t_c+t_0} \\
&\quad \left. b_{lk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), -\vec{\pi}'(\vec{q} \vec{q}_2), \vec{\pi}(\vec{q}_2 \vec{q}_0), \vec{\pi}'(\vec{q}_2 \vec{q}_0)) \right]
\end{aligned}$$

$$P_{13}^s P_{23}^s |1m_d\rangle |m_{10}\rangle \Big] . \quad (27)$$

Now we collect the spin parts and define

$$\begin{aligned} & C_{jlk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), \vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2\vec{q}_0), \vec{\pi}'(\vec{q}_2\vec{q}_0)) P_{12}^s P_{23}^s \\ & \equiv w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}(\vec{q}\vec{q}_2)) P_{12}^s P_{23}^s a_{lk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2\vec{q}_0), \vec{\pi}'(\vec{q}_2\vec{q}_0)) \\ & = w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}(\vec{q}\vec{q}_2)) a_{lk}(\vec{\sigma}(2), \vec{\sigma}(3), \vec{\sigma}(1), \vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2\vec{q}_0), \vec{\pi}'(\vec{q}_2\vec{q}_0)) P_{12}^s P_{23}^s , \end{aligned} \quad (28)$$

$$\begin{aligned} & D_{jlk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), \vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2\vec{q}_0), \vec{\pi}'(\vec{q}_2\vec{q}_0)) P_{12}^s P_{23}^s \\ & \equiv w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}(\vec{q}\vec{q}_2)) P_{12}^s P_{23}^s b_{lk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2\vec{q}_0), \vec{\pi}'(\vec{q}_2\vec{q}_0)) \\ & = w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}(\vec{q}\vec{q}_2)) b_{lk}(\vec{\sigma}(2), \vec{\sigma}(3), \vec{\sigma}(1), \vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2\vec{q}_0), \vec{\pi}'(\vec{q}_2\vec{q}_0)) P_{12}^s P_{23}^s , \end{aligned} \quad (29)$$

$$\begin{aligned} & E_{jlk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), \vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2\vec{q}_0), \vec{\pi}'(\vec{q}_2\vec{q}_0)) P_{13}^s P_{23}^s \\ & \equiv w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, -\vec{\pi}(\vec{q}\vec{q}_2)) P_{13}^s P_{23}^s a_{lk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), -\vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2\vec{q}_0), \vec{\pi}'(\vec{q}_2\vec{q}_0)) \\ & = w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, -\vec{\pi}(\vec{q}\vec{q}_2)) a_{lk}(\vec{\sigma}(3), \vec{\sigma}(1), \vec{\sigma}(2), -\vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2\vec{q}_0), \vec{\pi}'(\vec{q}_2\vec{q}_0)) P_{13}^s P_{23}^s , \end{aligned} \quad (30)$$

$$\begin{aligned} & F_{jlk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), \vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2\vec{q}_0), \vec{\pi}'(\vec{q}_2\vec{q}_0)) P_{13}^s P_{23}^s \\ & \equiv w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, -\vec{\pi}(\vec{q}\vec{q}_2)) P_{13}^s P_{23}^s b_{lk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), -\vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2\vec{q}_0), \vec{\pi}'(\vec{q}_2\vec{q}_0)) \\ & = w_j(\vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, -\vec{\pi}(\vec{q}\vec{q}_2)) b_{lk}(\vec{\sigma}(3), \vec{\sigma}(1), \vec{\sigma}(2), -\vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2\vec{q}_0), \vec{\pi}'(\vec{q}_2\vec{q}_0)) P_{13}^s P_{23}^s . \end{aligned} \quad (31)$$

Inserting these expressions into Eq. (27) yields

$$\begin{aligned} & \langle \vec{p}\vec{q} | \langle \gamma_a | t P G_0 t P | \Phi \rangle \\ & = F_{t_a t_c T_b} \sum_j \int d^3 q_2 \left[ t_{t_a T_a T_b}^{(j)}(\vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), E_q) G_0(\vec{\pi}'(\vec{q}\vec{q}_2) \vec{q}_2) \delta_{T_b T_c} \right. \\ & \quad \sum_{lk}^{(l)} (t_{t_c T_b T_0}^{(l)}(\vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2, \vec{q}_0), E_{q_2}) \phi_k(|\vec{\pi}'(\vec{q}_2, \vec{q}_0)|) F_{t_c t_0 T_0} \\ & \quad C_{jlk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), \vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2\vec{q}_0), \vec{\pi}'(\vec{q}_2\vec{q}_0)) P_{13}^s P_{23}^s \\ & + t_{t_c T_b T_0}^{(l)}(\vec{\pi}'(\vec{q}\vec{q}_2), -\vec{\pi}(\vec{q}_2, \vec{q}_0), E_{q_2}) \phi_k(|\vec{\pi}'(\vec{q}_2, \vec{q}_0)|) F_{t_c t_0 T_0} (-)^{t_c+t_0} \\ & \quad D_{jlk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), \vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2\vec{q}_0), \vec{\pi}'(\vec{q}_2\vec{q}_0)) \\ & + (-)^{t_a+t_c} t_{t_a T_a T_b}^{(j)}(\vec{p}, -\vec{\pi}(\vec{q}\vec{q}_2), E_q) G_0(\vec{\pi}'(\vec{q}\vec{q}_2) \vec{q}_2) \delta_{T_b T_c} \\ & \quad \sum_{lk}^{(l)} (t_{t_c T_b T_0}^{(l)}(-\vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2, \vec{q}_0), E_{q_2}) \phi_k(|\vec{\pi}'(\vec{q}_2, \vec{q}_0)|) F_{t_c t_0 T_0} \\ & \quad E_{jlk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), \vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2\vec{q}_0), \vec{\pi}'(\vec{q}_2\vec{q}_0)) \\ & + t_{t_c T_b T_0}^{(l)}(-\vec{\pi}'(\vec{q}\vec{q}_2), -\vec{\pi}(\vec{q}_2, \vec{q}_0), E_{q_2}) \phi_k(|\vec{\pi}'(\vec{q}_2, \vec{q}_0)|) F_{t_c t_0 T_0} (-)^{t_c+t_0} \\ & \quad E_{jlk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), \vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2\vec{q}_0), \vec{\pi}'(\vec{q}_2\vec{q}_0)) \\ & \left. P_{12}^s P_{23}^s \right] |1m_d\rangle |m_{10}\rangle . \end{aligned} \quad (32)$$

As will be shown for some examples in Appendix B the coefficients  $C_{jlk}$  from Eq. (28) together with the other coefficients from Eqs. (29) through (31) will depend on the following types of scalars. As an aside, one more term,  $\vec{\sigma}(i_1) \cdot \vec{q}_2(\vec{\sigma}(i_2) \times \vec{\sigma}(i_3)) \cdot \vec{q}_2$ , is expected to occur in  $C_{jlk}$  beyond the ones given in Appendix C:

$$\begin{aligned}
& C_{jlk}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), \vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2\vec{q}_0), \vec{\pi}'(\vec{q}_2\vec{q}_0)) \\
&= C_{jlk}^{(a)}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{q}, \vec{q}_0) + C_{jlk}^{(b)}(\vec{p}, \vec{q}, \vec{q}_0, \vec{q}_2) \\
&+ \sum_{i_1} C_{jlk; i_1}^{(c)}(\vec{\sigma}(i_2), \vec{\sigma}(i_3), \vec{p}, \vec{q}, \vec{q}_0) \vec{\sigma}(i_1) \cdot \vec{q}_2 \\
&+ \sum_{i_1 i_2} C_{jlk; i_1 i_2}^{(d)}(\vec{\sigma}(i_3), \vec{p}, \vec{q}, \vec{q}_0) \vec{\sigma}(i_1) \cdot \vec{q}_2 \vec{\sigma}(i_2) \cdot \vec{q}_2 \\
&+ C_{jlk}^{(e)}(\vec{p}, \vec{q}, \vec{q}_0) \vec{\sigma}(i_1) \cdot \vec{q}_2 \vec{\sigma}(i_2) \cdot \vec{q}_2 \sigma(i_3) \cdot \vec{q}_2 \\
&+ \sum_{i_1 \neq i_2} C_{jlk; i_1 i_2}^{(f)}(\vec{\sigma}(i_3), \vec{p}, \vec{q}, \vec{q}_0) \vec{\sigma}(i_1) \times \vec{\sigma}(i_2) \cdot \vec{q}_2 \\
&+ \sum_{i_1 \neq i_2 \neq i_3} C_{jlk; i_1 i_2 i_3}^{(g)}(\vec{p}, \vec{q}, \vec{q}_0), \vec{\sigma}(i_1) \cdot \vec{q}_2(\vec{\sigma}(i_2) \times \vec{\sigma}(i_3)) \cdot \vec{q}_2, \tag{33}
\end{aligned}$$

where  $i_1, i_2, i_3 = 1, 2, 3$  and cyclical permutations thereof.

It is sufficient to consider the first integral in Eq. (27) going along with the coefficients  $C_{jlk}$ . We insert Eq. (33):

$$\begin{aligned}
& \langle \vec{p}\vec{q} | \langle \gamma_a | t P G_0 t P | \Phi \rangle^{(1)} \\
&= F_{t_a t_c T_b} \delta_{T_b T_c} F_{t_c t_0 T_0} \sum_j \sum_{lk} \left[ C_{jlk}^{(a)}(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{q}, \vec{q}_0) \right. \\
&\quad \int d^3 q_2 t_{t_a T_a T_b}^{(j)}(\vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), E_q) G_0(\vec{\pi}'(\vec{q}\vec{q}_2) \vec{q}_2) \\
&\quad t_{t_c T_b T_0}^{(l)}(\vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2, \vec{q}_0), E_{q_2}) \phi_k(|\vec{\pi}'(\vec{q}_2, \vec{q}_0)|) \\
&+ \int d^3 q_2 t_{t_a T_a T_b}^{(j)}(\vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), E_q) G_0(\vec{\pi}'(\vec{q}\vec{q}_2) \vec{q}_2) \\
&\quad t_{t_c T_b T_0}^{(l)}(\vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2, \vec{q}_0), E_{q_2}) \phi_k(|\vec{\pi}'(\vec{q}_2, \vec{q}_0)|) C_{jlk}^{(b)}(\vec{p}, \vec{q}, \vec{q}_0, \vec{q}_2) \\
&+ \sum_{i_1} C_{jlk; i_1}^{(c)}(\vec{\sigma}(i_2), \vec{\sigma}(i_3), \vec{p}, \vec{q}, \vec{q}_0) \int d^3 q_2 t_{t_a T_a T_b}^{(j)}(\vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), E_q) G_0(\vec{\pi}'(\vec{q}\vec{q}_2) \vec{q}_2) \\
&\quad t_{t_c T_b T_0}^{(l)}(\vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2, \vec{q}_0), E_{q_2}) \phi_k(|\vec{\pi}'(\vec{q}_2, \vec{q}_0)|) \vec{\sigma}(i_1) \cdot \vec{q}_2 \\
&+ \sum_{i_1 i_2} C_{jlk; i_1 i_2}^{(d)}(\vec{\sigma}(i_3), \vec{p}, \vec{q}, \vec{q}_0) \int d^3 q_2 t_{t_a T_a T_b}^{(j)}(\vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), E_q) G_0(\vec{\pi}'(\vec{q}\vec{q}_2) \vec{q}_2) \\
&\quad t_{t_c T_b T_0}^{(l)}(\vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2, \vec{q}_0), E_{q_2}) \phi_k(|\vec{\pi}'(\vec{q}_2, \vec{q}_0)|) \vec{\sigma}(i_1) \cdot \vec{q}_2 \vec{\sigma}(i_2) \cdot \vec{q}_2 \\
&+ C_{jlk}^{(e)}(\vec{p}, \vec{q}, \vec{q}_0) \int d^3 q_2 t_{t_a T_a T_b}^{(j)}(\vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), E_q) G_0(\vec{\pi}'(\vec{q}\vec{q}_2) \vec{q}_2) \\
&\quad t_{t_c T_b T_0}^{(l)}(\vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2, \vec{q}_0), E_{q_2}) \phi_k(|\vec{\pi}'(\vec{q}_2, \vec{q}_0)|) \vec{\sigma}(i_1) \cdot \vec{q}_2 \vec{\sigma}(i_2) \cdot \vec{q}_2 \sigma(i_3) \cdot \vec{q}_2 \\
&+ \sum_{i_1 \neq i_2} C_{jlk; i_1 i_2}^{(f)}(\vec{\sigma}(i_3), \vec{p}, \vec{q}, \vec{q}_0) \int d^3 q_2 t_{t_a T_a T_b}^{(j)}(\vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), E_q) G_0(\vec{\pi}'(\vec{q}\vec{q}_2) \vec{q}_2) \\
&\quad t_{t_c T_b T_0}^{(l)}(\vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2, \vec{q}_0), E_{q_2}) \phi_k(|\vec{\pi}'(\vec{q}_2, \vec{q}_0)|) \vec{\sigma}(i_1) \times \vec{\sigma}(i_2) \cdot \vec{q}_2
\end{aligned}$$

$$\begin{aligned}
& + \sum_{i_1 \neq i_2 \neq i_3} C_{jlk;i_1 i_2 i_3}^{(g)}(\vec{p}, \vec{q}, \vec{q}_0) \int d^3 q_2 t_{t_a T_a T_b}^{(j)}(\vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), E_q) G_0(\vec{\pi}'(\vec{q}\vec{q}_2) \vec{q}_2) \\
& t_{t_c T_b T_0}^{(l)}(\vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2, \vec{q}_0), E_{q_2}) \phi_k(|\vec{\pi}'(\vec{q}_2, \vec{q}_0)|) \vec{\sigma}(i_1) \cdot \vec{q}_2 (\vec{\sigma}(i_2) \times \vec{\sigma}(i_3)) \cdot \vec{q}_2 \Big] \\
& P_{13}^s P_{23}^s |1m_d\rangle |m_{10}\rangle .
\end{aligned} \tag{34}$$

The integrals in Eq. (34) are of the following form

$$X_1 = \int d^3 q_2 Y , \tag{35}$$

$$X_2 = \int d^3 q_2 Y \vec{\sigma}(i) \cdot \vec{q}_2 , \tag{36}$$

$$X_3 = \int d^3 q_2 Y \vec{\sigma}(i) \cdot \vec{q}_2 \vec{\sigma}(j) \cdot \vec{q}_2 , \tag{37}$$

$$X_4 = \int d^3 q_2 Y \vec{\sigma}(i) \cdot \vec{q}_2 \vec{\sigma}(j) \cdot \vec{q}_2 \vec{\sigma}(k) \cdot \vec{q}_2 , \tag{38}$$

$$X_5 = \int d^3 q_2 Y \vec{\sigma}(i) \times \vec{\sigma}(j) \cdot \vec{q}_2 , \tag{39}$$

$$X_6 = \int d^3 q_2 Y \vec{\sigma}(i_1) \cdot \vec{q}_2 (\vec{\sigma}(i_2) \times \vec{\sigma}(i_3)) \cdot \vec{q}_2 , \tag{40}$$

where  $Y$  represents functions with different dependencies on scalar quantities. For instance in the first integral in Eq. (34) one has

$$\begin{aligned}
Y &= t_{t_a T_a T_b}^{(j)}(\vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), E_q) G_0(\vec{\pi}'(\vec{q}\vec{q}_2) \vec{q}_2) \\
& t_{t_c T_b T_0}^{(l)}(\vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2, \vec{q}_0), E_{q_2}) \phi_k(|\vec{\pi}'(\vec{q}_2, \vec{q}_0)|) .
\end{aligned} \tag{41}$$

As detailed in the Appendix C its angular dependence is

$$Y = Y(\hat{p} \cdot \hat{q}, \hat{p} \cdot \hat{q}_2, \hat{q} \cdot \hat{q}_2, \hat{q} \cdot \hat{q}_0, \hat{q}_2 \cdot \hat{q}_0) . \tag{42}$$

The remaining integrals will be treated as exemplified for  $X_2$ . The integral has the form

$$X_2 = \sigma_s(i) \int d^3 q_2 Y q_{2s} . \tag{43}$$

If  $Y$  does not depend on  $\vec{q}_2$  then  $X_2 = 0$ . Otherwise, due to the scalar nature of  $Y$ , one must have the following structure

$$\int d^3 q_2 Y q_{2,s} = \alpha p_s + \beta q_s + \gamma q_{0s} , \tag{44}$$

and therefore

$$X_2 = \alpha \vec{\sigma}(i) \cdot \vec{p} + \beta \vec{\sigma}(i) \cdot \vec{q} + \gamma \vec{\sigma}(i) \cdot \vec{q}_0 . \tag{45}$$

The scalars  $\alpha, \beta, \gamma$  are then determined by multiplying Eq. (44) by  $p_s$ ,  $q_s$ , and  $q_{0s}$ , respectively. This leads to three equations

$$\int d^3 q_2 Y \vec{p} \cdot \vec{q}_2 = \alpha p^2 + \beta \vec{p} \cdot \vec{q} + \gamma \vec{p} \cdot \vec{q}_0 , \quad (46)$$

$$\int d^3 q_2 Y \vec{q} \cdot \vec{q}_2 = \alpha \vec{q} \cdot \vec{p} + \beta q^2 + \gamma \vec{q} \cdot \vec{q}_0 , \quad (47)$$

$$\int d^3 q_2 Y \vec{q}_0 \cdot \vec{q}_2 = \alpha \vec{q}_0 \cdot \vec{p} + \beta \vec{q}_0 \cdot \vec{q} + \gamma q_0^2 , \quad (48)$$

for  $\alpha, \beta, \gamma$ . The three integrals have to be determined numerically.

Correspondingly, despite being more involved,  $X_3$  can be determined

$$X_3 = \sigma_s(i) \sigma_t(j) \int d^3 q_2 Y q_{2s} q_{2t} . \quad (49)$$

One has three external momenta in the function  $Y$ , namely  $\vec{p}, \vec{q}$ , and  $\vec{q}_0$ . Thus, there are at most 9 possibilities:

$$\int d^3 q_2 Y q_{2s} q_{2t} = \sum_{L=1}^9 A_L Q_{Ls} Q'_{Lt} , \quad (50)$$

where the  $Q_{Ls} Q'_{Lt}$  belong to the set

$$Q_{Ls} Q'_{Lt} = [p_s p_t, p_s q_t, p_s q_{0t}, q_s p_t, q_s q_t, q_s q_{0t}, q_{0s} p_t, q_{0s} q_t, q_{0s} q_{0t}] . \quad (51)$$

By appropriate multiplications from the left one will have nine equations for the coefficients  $A_L$ , generated by 9 corresponding integrals. Thus  $X_3$  takes the form

$$X_3 = \sum_{L=1}^9 A_L \vec{\sigma}(i) \cdot \vec{Q}_L \vec{\sigma}(j) \cdot \vec{Q}'_L . \quad (52)$$

Furthermore,

$$X_4 = \sigma_s(i) \sigma_t(j) \sigma_u(k) \int d^3 q_2 Y q_{2s} q_{2t} q_{2u} . \quad (53)$$

Now we have 27 possibilities and consequently 27 equations need to be solved with the result

$$X_4 = \sum_{L=1}^{27} B_L \vec{\sigma}(i) \cdot \vec{Q}_L \vec{\sigma}(j) \cdot \vec{Q}'_L \vec{\sigma}(k) \cdot \vec{Q}''_L , \quad (54)$$

where  $Q_{Ls} Q'_{Lt} Q''_{Lu}$  are out of the set

$$Q_{Ls} Q'_{Lt} Q''_{Lu} = [p_s p_t p_u, p_s p_t q_u, p_s p_t q_{0u}, \dots] . \quad (55)$$

The appropriate 27 integrals need to be determined numerically. Finally,  $X_5$  can be handled like  $X_2$ , and  $X_6$  like  $X_3$ . All the integrals occurring in Eq. (34) and the ones following from that are of the type

$$H \equiv \int d^3q_2 t_{t_a T_a T_b}^{(j)}(\vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), E_q) G_0(\vec{\pi}'(\vec{q}\vec{q}_2)\vec{q}_2) t_{t_c T_b T_0}^{(l)}(\vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2, \vec{q}_0), E_{q_2}) \phi_k(|\vec{\pi}'(\vec{q}_2, \vec{q}_0)|) f, \quad (56)$$

where  $f$  consist of scalar products of the momenta  $\vec{p}, \vec{q}, \vec{q}_0$ , and  $\vec{q}_2$ .

We use the decomposition of the NN t-matrix from Eq. (20) in the pd isospin space and the deuteron pole structure from Eq. (23). Then we obtain

$$\begin{aligned} H &= \int d^3q_2 G_0(\vec{\pi}'(\vec{q}\vec{q}_2)\vec{q}_2) \phi_k(|\vec{\pi}'(\vec{q}_2, \vec{q}_0)|) f \\ &\quad \left( \delta_{t_a 0} t_{0\frac{1}{2}\frac{1}{2}}^{(j)}(\vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), E_q) + \delta_{t_a 1} t_{1T_a T_b}^{(j)}(\vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), E_q) \right) \\ &\quad \left( \delta_{t_c 0} t_{0\frac{1}{2}\frac{1}{2}}^{(l)}(\vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2, \vec{q}_0), E_{q_2}) + \delta_{t_c 1} t_{1T_b T_b}^{(l)}(\vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2, \vec{q}_0), E_{q_2}) \right) \\ &= \int d^3q_2 G_0(\vec{\pi}'(\vec{q}\vec{q}_2)\vec{q}_2) \phi_k(|\vec{\pi}'(\vec{q}_2, \vec{q}_0)|) \\ &\quad \left( \delta_{t_a 0} \frac{\hat{t}_{np}^{(00,j)}(\vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), E_q)}{E_q + i\epsilon - E_d} + \delta_{t_a 1} t_{1T_a T_b}^{(j)}(\vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), E_q) \right) \\ &\quad \left( \delta_{t_c 0} \frac{\hat{t}_{np}^{(00,l)}(\vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2, \vec{q}_0), E_{q_2})}{E_{q_2} + i\epsilon - E_d} + \delta_{t_c 1} t_{1T_b T_b}^{(l)}(\vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2, \vec{q}_0), E_{q_2}) \right). \quad (57) \end{aligned}$$

In numerical implementation one can either follow the standard path [1] dealing with the free propagator singularity and leading to moving logarithmic singularities, whose treatment is well controlled and documented in [1, 13] or one applies the new way [20, 21] which avoids that complication totally. We suggest the second path, which for the convenience of the reader is detailed again in the Appendix D.

Summarizing, the second order amplitude of Eq. (32) using all information given above will have the structure

$$\langle \vec{p}\vec{q} | \langle \gamma_a | t P G_0 t P | \Phi \rangle = \sum_f S_f(\vec{p}, \vec{q}, \vec{q}_0) O_f(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{q}, \vec{q}_0) P_f | 1m_d \rangle | m_{10} \rangle, \quad (58)$$

where  $O_f$  are spin-momentum dependent scalar operators and  $S_f$  only momentum dependent scalar functions, all depending just on the external momenta  $\vec{p}, \vec{q}$ , and  $\vec{q}_0$ . Further  $P_f$  is either 1,  $P_{12}^s P_{23}^s$  or  $P_{13}^s P_{23}^s$ .

This form will be the input for the next order in Eq. (4), namely  $t P G_0(t P G_0 t P | \Phi)$  where the kernel  $t P G_0$  is exactly treated as above and the first order term  $t P | \Phi$  is replaced by that second

order form. Obviously this will go on like that and the total amplitude  $\langle \vec{p}\vec{q} | \langle \gamma_a | T | \Phi \rangle$  will appear in the form of the right hand side of Eq. (58).

It remains to present the calculation of the observables, which is described for the example of the breakup cross section in the next section.

## V. THE ND BREAKUP CROSS SECTION

The full breakup amplitude according to Eq. (3), projected on spin states and using Eq. (8) is given by

$$\begin{aligned}
& \langle m_1 | \langle m_2 | \langle m_3 | \langle \vec{p}\vec{q} | \langle \gamma_a | (1 + P) T | \Phi \rangle \\
&= \langle m_1 | \langle m_2 | \langle m_3 | \langle \vec{p}\vec{q} | \langle \gamma_a | T | \Phi \rangle + \langle m_1 | \langle m_2 | \langle m_3 | \langle \vec{p}\vec{q} | \langle \gamma_a | P T | \Phi \rangle \\
&= \langle m_1 | \langle m_2 | \langle m_3 | \langle \vec{p}\vec{q} | \langle \gamma_a | T | \Phi \rangle \\
&+ \sum_b F_{t_a t_b T_a} \delta_{T_a T_b} \langle m_1 | \langle m_2 | \langle m_3 | \langle \vec{p}\vec{q} | (P_{12}^{sm} P_{23}^{sm} + (-)^{t_a + t_b} P_{13}^{sm} P_{23}^{sm}) \langle \gamma_b | T | \Phi \rangle . \quad (59)
\end{aligned}$$

The simplest approach is to apply the permutations to the left, using

$$\langle \vec{p}\vec{q} | P_{13}^m P_{23}^m = \langle -\frac{1}{2}\vec{p} + \frac{3}{4}\vec{q}, -\vec{p} - \frac{1}{2}\vec{q} | \equiv \langle \vec{p}^{(1)} \vec{q}^{(1)} | , \quad (60)$$

$$\langle \vec{p}\vec{q} | P_{12}^m P_{23}^m = \langle -\frac{1}{2}\vec{p} - \frac{3}{4}\vec{q}, \vec{p} - \frac{1}{2}\vec{q} | \equiv \langle \vec{p}^{(2)} \vec{q}^{(2)} | . \quad (61)$$

Therefore,

$$\begin{aligned}
& \langle m_1 | \langle m_2 | \langle m_3 | \langle \vec{p}\vec{q} | \langle \gamma_a | (1 + P) T | \Phi \rangle \\
&= \langle m_1 | \langle m_2 | \langle m_3 | \langle \vec{p}\vec{q} | \langle \gamma_a | T | \Phi \rangle \\
&+ \sum_b F_{t_a t_b T_a} \delta_{T_a T_b} \langle m_1 | \langle m_2 | \langle m_3 | P_{12}^s P_{23}^s \langle \vec{p}^{(2)} \vec{q}^{(2)} | \langle \gamma_b | T | \Phi \rangle \\
&+ \sum_b F_{t_a t_b T_a} \delta_{T_a T_b} (-)^{t_a + t_b} \langle m_1 | \langle m_2 | \langle m_3 | P_{13}^s P_{23}^s \langle \vec{p}^{(1)} \vec{q}^{(1)} | \langle \gamma_b | T | \Phi \rangle . \quad (62)
\end{aligned}$$

Using now the general form of the right hand side of Eq. (58) one obtains

$$\begin{aligned}
& \langle m_1 | \langle m_2 | \langle m_3 | \langle \vec{p}\vec{q} | \langle \gamma_a | (1 + P) T | \Phi \rangle \\
&= \langle m_1 | \langle m_2 | \langle m_3 | \sum_f S_f(\vec{p}, \vec{q}, \vec{q}_0) O_f(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{q}, \vec{q}_0) P_f | 1m_d \rangle | m_{10} \rangle \\
&+ \sum_b F_{t_a t_b T_a} \delta_{T_a T_b} \langle m_1 | \langle m_2 | \langle m_3 | P_{12}^s P_{23}^s \\
&\quad \sum_f S_f(\vec{p}^{(2)}, \vec{q}^{(2)}, \vec{q}_0) O_f(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}^{(2)}, \vec{q}^{(2)}, \vec{q}_0) P_f | 1m_d \rangle | m_{10} \rangle \\
&+ \sum_b F_{t_a t_b T_a} \delta_{T_a T_b} (-)^{t_a + t_b} \langle m_1 | \langle m_2 | \langle m_3 | P_{13}^s P_{23}^s
\end{aligned}$$

$$\begin{aligned}
& \sum_f S_f(\vec{p}^{(1)}, \vec{q}^{(1)}, \vec{q}_0) O_f(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}^{(1)}, \vec{q}^{(1)}, \vec{q}_0) P_f |1m_d\rangle |m_{10}\rangle \\
& \equiv \sum_\alpha f_\alpha(\vec{p}, \vec{q}, \vec{q}_0) \langle m_1 m_2 m_3 | o_\alpha(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{q}, \vec{q}_0) P_\alpha |1m_d\rangle |m_{10}\rangle .
\end{aligned} \tag{63}$$

As shown in Ref. [1], the density matrix for the final state is given by

$$(\rho_f)_{ij} = \sum_{kl} N_{ik}(\rho_i)_{kl} N_{lj}^\dagger , \tag{64}$$

where

$$N_{ij} = \langle \Lambda_i | \langle \vec{p} \vec{q} | U_0 | \phi \rangle | \vec{q}_0 \rangle | \lambda_j \rangle , \tag{65}$$

is the expression of Eq. (63), where the set of spin magnetic quantum numbers is given by

$$\begin{aligned}
|\Lambda_i\rangle &= |m_1\rangle |m_2\rangle |m_3\rangle , \\
|\lambda_j\rangle &= |m_{10}\rangle |1m_d\rangle ,
\end{aligned} \tag{66}$$

Furthermore,  $\rho_i$  is the density matrix for the initial state,

$$\rho_i = \frac{1}{6} Tr(\rho_i) \sum_\nu S^\nu \langle S^\nu \rangle_i , \tag{67}$$

with  $[S^\nu]$  being the complete set of 2N spin matrices [1]. For an unpolarized initial state one has

$$\rho_i = \frac{1}{6} Tr(\rho_i) 1_{2 \times 2} \times 1_{3 \times 3} . \tag{68}$$

With this, the unpolarized breakup cross section is given up to a phase-space factor as

$$\begin{aligned}
\sigma &\sim \frac{Tr(\rho_f)}{Tr(\rho_i)} = \frac{1}{6} \sum_i \sum_k N_{ik} N_{ik}^* \\
&= \frac{1}{6} \sum_i \sum_k |\langle \Lambda_i | \langle \vec{p} \vec{q} | U_0 | \phi \rangle | \vec{q}_0 \rangle | \lambda_k \rangle|^2 \\
&= \frac{1}{6} \sum_{m_1 m_2 m_3} \sum_{m_d m_{10}} \langle m_1 m_2 m_3 | \langle \vec{p} \vec{q} | U_0 | \phi \rangle | \vec{q}_0 \rangle | 1m_d \rangle | m_{10} \rangle \\
&\quad \langle 1m_d | \langle m_{10} | \langle \vec{q}_0 | \langle \phi | U_0^\dagger | \vec{p} \vec{q} \rangle | m_1 m_2 m_3 \rangle .
\end{aligned} \tag{69}$$

Using now the final form in Eq. (63) we obtain

$$\begin{aligned}
\sigma &\sim \frac{1}{6} \sum_{m_1 m_2 m_3} \sum_{m_d m_{10}} \sum_\alpha f_\alpha(\vec{p}, \vec{q}, \vec{q}_0) \\
&\quad \langle m_1 m_2 m_3 | o_\alpha(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{q}, \vec{q}_0) P_\alpha | 1m_d \rangle | m_{10} \rangle \\
&\quad \sum_\beta f_\beta^*(\vec{p}, \vec{q}, \vec{q}_0) \langle 1m_d | \langle m_{10} | P_\beta o_\beta^\dagger(\vec{\sigma}(1), \vec{\sigma}(2), \vec{\sigma}(3), \vec{p}, \vec{q}, \vec{q}_0) | m_1 m_2 m_3 \rangle \\
&= \frac{1}{6} \sum_{\alpha, \beta} f_\alpha f_\beta^* \sum_{m_1 m_2 m_3} \langle m_1 m_2 m_3 | o_\alpha P_\alpha P_\beta o_\beta^\dagger | m_1 m_2 m_3 \rangle .
\end{aligned} \tag{70}$$



## VI. SUMMARY AND CONCLUSIONS

We extended the recently developed formalism for a new treatment of two- and three-nucleon bound states in three dimensions to the realm of nucleon-deuteron scattering. The aim is to formulate the momentum space Faddeev equations in such a fashion that the equations are reduced to spin independent scalar functions in the same spirit as already worked out for the 2N and 3N bound states. We use the original, most general structure of NN forces, which is built out of scalar operators of spin- and momentum-vectors plus scalar functions that only depend on momenta. This structure carries over to the NN t-operator which is a central building block in the Faddeev scheme. Generating the multiple scattering series for the Faddeev equations we arrive at the general structure of the multiple scattering terms in the form of scalar spin-momentum dependent operators and momentum dependent scalar functions. Since our formulation only depends on operator and momentum vectors, there are no intrinsic limitations on the energy range in which it can be applied. This is contrast to the partial wave projected form of the momentum space Faddeev equations.

Since three nucleon forces appear naturally in the form of scalar operators in spin- and momentum-vectors and scalar functions depending only on momenta, the present formulation can be extended to include also 3N forces in a straightforward manner. This feature is especially important in view of the chiral perturbation theory approach, where the multitude of 3N forces at N<sup>3</sup>LO appears to be most efficiently treated directly in this three-dimensional formulation without having to expand these forces into a partial wave basis.

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## APPENDIX A: THE SCALAR EXPRESSIONS $a_{jk}$ AND $b_{jk}$

Since the scalar expressions  $a_{jk}$  and  $b_{jk}$  from Eqs. (16) and (17) have to be determined only once and their number is not too large, we provide all of them:

$$a_{11} = 1 \quad (\text{A1})$$

$$a_{12} = -\frac{1}{3}\pi'^2 + (\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(3) \cdot \vec{\pi}') \quad (\text{A2})$$

$$a_{21} = \vec{\sigma}(2) \cdot \vec{\sigma}(3) \quad (\text{A3})$$

$$\begin{aligned} a_{22} &= (\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(2) \cdot \vec{\pi}') \\ &\quad - i(\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(2) \times \vec{\sigma}(3)) \cdot \vec{\pi}' \\ &\quad - \frac{1}{3}\pi'^2(\vec{\sigma}(2) \cdot \vec{\sigma}(3)) \end{aligned} \quad (\text{A4})$$

$$a_{31} = (\vec{\sigma}(2) + \vec{\sigma}(3)) \cdot (\vec{p} \times \vec{\pi}) \quad (\text{A5})$$

$$\begin{aligned} a_{32} &= \frac{3}{4}\vec{q}_0 \cdot (\vec{p} \times \vec{q})(\vec{\sigma}(1) \cdot \vec{\pi}') \\ &\quad - i(\vec{\pi}' \cdot \vec{\pi})(\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(3)) \cdot \vec{p} \\ &\quad + i(\vec{p} \cdot \vec{\pi}')(\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(3) \cdot \vec{\pi}) \\ &\quad + (\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(2) \cdot (\vec{p} \times \vec{\pi}))(\vec{\sigma}(3) \cdot \vec{\pi}') \\ &\quad - \frac{1}{3}\pi'^2(\vec{\sigma}(2) + \vec{\sigma}(3)) \cdot (\vec{p} \times \vec{\pi}) \end{aligned} \quad (\text{A6})$$

$$a_{41} = \vec{\sigma}(2) \cdot (\vec{p} \times \vec{\pi})\vec{\sigma}(3) \cdot (\vec{p} \times \vec{\pi}) \quad (\text{A7})$$

$$\begin{aligned} a_{42} &= \frac{3}{4}(\vec{p} \times \vec{q}) \cdot \vec{q}_0(\vec{\sigma}(1) \cdot \vec{\pi}')\vec{\sigma}(2) \cdot (\vec{p} \times \vec{\pi}) \\ &\quad - i(\vec{\pi}' \cdot \vec{\pi})(\vec{\sigma}(1) \cdot \vec{\pi}')\vec{\sigma}(2) \cdot (\vec{p} \times \vec{\pi})(\vec{\sigma}(3) \cdot \vec{p}) \\ &\quad + i(\vec{p} \cdot \vec{\pi}')(\vec{\sigma}(1) \cdot \vec{\pi}')\vec{\sigma}(2) \cdot (\vec{p} \times \vec{\pi})(\vec{\sigma}(3) \cdot \vec{\pi}) \\ &\quad - \frac{1}{3}\pi'^2\vec{\sigma}(2) \cdot (\vec{p} \times \vec{\pi})\vec{\sigma}(3) \cdot (\vec{p} \times \vec{\pi}) \end{aligned} \quad (\text{A8})$$

$$a_{51} = (\vec{\sigma}(2) \cdot (\vec{p} + \vec{\pi}))(\vec{\sigma}(3) \cdot (\vec{p} + \vec{\pi})) \quad (\text{A9})$$

$$\begin{aligned} a_{52} &= ((\vec{p} + \vec{\pi}) \cdot \vec{\pi}')(\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(2) \cdot (\vec{p} + \vec{\pi})) \\ &\quad + i(\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(2) \cdot (\vec{p} + \vec{\pi}))(\vec{\sigma}(3) \cdot (\vec{p} \times \vec{\pi}')) \\ &\quad + i\frac{3}{4}(\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(2) \cdot (\vec{p} + \vec{\pi}))(\vec{\sigma}(3) \cdot (\vec{q} \times \vec{q}_0)) \\ &\quad - \frac{1}{3}\pi'^2(\vec{\sigma}(2) \cdot (\vec{p} + \vec{\pi}))(\vec{\sigma}(3) \cdot (\vec{p} + \vec{\pi})) \end{aligned} \quad (\text{A10})$$

$$a_{61} = (\vec{\sigma}(2) \cdot (\vec{p} - \vec{\pi}))(\vec{\sigma}(3) \cdot (\vec{p} - \vec{\pi})) \quad (\text{A11})$$

$$\begin{aligned} a_{62} &= ((\vec{p} - \vec{\pi}) \cdot \vec{\pi}')(\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(2) \cdot (\vec{p} - \vec{\pi})) \\ &\quad + i(\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(2) \cdot (\vec{p} - \vec{\pi}))(\vec{\sigma}(3) \cdot (\vec{p} \times \vec{\pi}')) \\ &\quad - i\frac{3}{4}(\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(2) \cdot (\vec{p} - \vec{\pi}))(\vec{\sigma}(3) \cdot (\vec{q} \times \vec{q}_0)) \\ &\quad - \frac{1}{3}\pi'^2(\vec{\sigma}(2) \cdot (\vec{p} - \vec{\pi}))(\vec{\sigma}(3) \cdot (\vec{p} - \vec{\pi})) \end{aligned} \quad (\text{A12})$$

$$b_{11} = 1 \quad (\text{A13})$$

$$b_{12} = -\frac{1}{3}\pi'^2 + (\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(2) \cdot \vec{\pi}') \quad (\text{A14})$$

$$b_{21} = \vec{\sigma}(2) \cdot \vec{\sigma}(3) \quad (\text{A15})$$

$$\begin{aligned} b_{22} &= (\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(3) \cdot \vec{\pi}') \\ &+ i(\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(2) \times \vec{\sigma}(3)) \cdot \vec{\pi}' \\ &- \frac{1}{3}\pi'^2(\vec{\sigma}(2) \cdot \vec{\sigma}(3)) \end{aligned} \quad (\text{A16})$$

$$b_{31} = -(\vec{\sigma}(2) + \vec{\sigma}(3)) \cdot \vec{p} \times \vec{\pi} \quad (\text{A17})$$

$$\begin{aligned} b_{32} &= -\frac{3}{4}(\vec{p} \times \vec{q}) \cdot \vec{q}_0(\vec{\sigma}(1) \cdot \vec{\pi}') \\ &+ i(\vec{\pi} \cdot \vec{\pi}')(\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(2) \cdot \vec{p}) \\ &- i(\vec{p} \cdot \vec{\pi}')(\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(2) \cdot \vec{\pi}) \\ &- (\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(2) \cdot \vec{\pi}')(\vec{\sigma}(3) \cdot (\vec{p} \times \vec{\pi})) \\ &+ \frac{1}{3}\pi'^2(\vec{\sigma}(2) + \vec{\sigma}(3)) \cdot (\vec{p} \times \vec{\pi}) \end{aligned} \quad (\text{A18})$$

$$b_{41} = \vec{\sigma}(2) \cdot (\vec{p} \times \vec{\pi})\vec{\sigma}(3) \cdot (\vec{p} \times \vec{\pi}) \quad (\text{A19})$$

$$\begin{aligned} b_{42} &= \frac{3}{4}(\vec{p} \times \vec{q}) \cdot \vec{q}_0(\vec{\sigma}(1) \cdot \vec{\pi}')\vec{\sigma}(3) \cdot (\vec{p} \times \vec{\pi}) \\ &- i(\vec{\pi} \cdot \vec{\pi}')(\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(2) \cdot \vec{p})\vec{\sigma}(3) \cdot (\vec{p} \times \vec{\pi}) \\ &+ i(\vec{p} \cdot \vec{\pi}')(\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(2) \cdot \vec{\pi})\vec{\sigma}(3) \cdot (\vec{p} \times \vec{\pi}) \\ &- \frac{1}{3}\pi'^2\vec{\sigma}(2) \cdot (\vec{p} \times \vec{\pi})\vec{\sigma}(3) \cdot (\vec{p} \times \vec{\pi}) \end{aligned} \quad (\text{A20})$$

$$b_{51} = (\vec{\sigma}(2) \cdot (\vec{p} - \vec{\pi}))(\vec{\sigma}(3) \cdot (\vec{p} - \vec{\pi})) \quad (\text{A21})$$

$$\begin{aligned} b_{52} &= (\vec{p} - \vec{\pi}) \cdot \vec{\pi}'(\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(3) \cdot (\vec{p} - \vec{\pi})) \\ &+ i(\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(2) \cdot (\vec{p} \times \vec{\pi}'))(\vec{\sigma}(3) \cdot (\vec{p} - \vec{\pi})) \\ &- i\frac{3}{4}(\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(2) \cdot (\vec{q} \times \vec{q}_0))(\vec{\sigma}(3) \cdot (\vec{p} - \vec{\pi})) \\ &- \frac{1}{3}\pi'^2(\vec{\sigma}(2) \cdot (\vec{p} - \vec{\pi}))(\vec{\sigma}(3) \cdot (\vec{p} - \vec{\pi})) \end{aligned} \quad (\text{A22})$$

$$b_{61} = (\vec{\sigma}(2) \cdot (\vec{p} + \vec{\pi}))(\vec{\sigma}(3) \cdot (\vec{p} + \vec{\pi})) \quad (\text{A23})$$

$$\begin{aligned} b_{62} &= ((\vec{p} + \vec{\pi}) \cdot \vec{\pi}')(\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(3) \cdot (\vec{p} + \vec{\pi})) \\ &+ i(\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(2) \cdot (\vec{p} \times \vec{\pi}'))(\vec{\sigma}(3) \cdot (\vec{p} + \vec{\pi})) \\ &+ \frac{3}{4}i(\vec{\sigma}(1) \cdot \vec{\pi}')(\vec{\sigma}(2) \cdot (\vec{q} \times \vec{q}_0))(\vec{\sigma}(3) \cdot (\vec{p} + \vec{\pi})) \\ &- \frac{1}{3}\pi'^2(\vec{\sigma}(2) \cdot (\vec{p} + \vec{\pi}))(\vec{\sigma}(3) \cdot (\vec{p} + \vec{\pi})) \end{aligned} \quad (\text{A24})$$

## APPENDIX B: EXAMPLES FOR THE COEFFICIENTS $C_{jlk}$

Again the coefficients  $C_{jlk}$ ,  $D_{jlk}$ ,  $E_{jlk}$  and  $F_{jlk}$  defined in Eqs. (28)-(31) have to be calculated only once. In the following we provide some examples, inserting Eqs. (10) for  $\vec{\pi}$  and  $\vec{\pi}'$  and removing the double occurrences of the same  $\vec{\sigma}(i)$ :

$$C_{111} = 1 \quad (B1)$$

$$\begin{aligned} C_{112} = & -\frac{1}{3}(\vec{q}_2 + \frac{1}{2}\vec{q}_0)^2 \\ & + (\vec{\sigma}(2) \cdot \vec{q}_2)(\vec{\sigma}(1) \cdot \vec{q}_2) + (\vec{\sigma}(2) \cdot \vec{q}_2)(\vec{\sigma}(1) \cdot \frac{1}{2}\vec{q}_0) \\ & + (\vec{\sigma}(2) \cdot \frac{1}{2}\vec{q}_0)(\vec{\sigma}(1) \cdot \vec{q}_2) + (\vec{\sigma}(2) \cdot \frac{1}{2}\vec{q}_0)(\vec{\sigma}(1) \cdot \frac{1}{2}\vec{q}_0) \end{aligned} \quad (B2)$$

$$C_{121} = \vec{\sigma}(3) \cdot \vec{\sigma}(1) \quad (B3)$$

$$\begin{aligned} C_{122} = & (\vec{\sigma}(2) \cdot \vec{q}_2)(\vec{\sigma}(3) \cdot \vec{q}_2) + (\vec{\sigma}(2) \cdot \vec{q}_2)(\vec{\sigma}(3) \cdot \frac{1}{2}\vec{q}_0) \\ & + (\vec{\sigma}(2) \cdot \frac{1}{2}\vec{q}_0)(\vec{\sigma}(3) \cdot \vec{q}_2) + (\vec{\sigma}(2) \cdot \frac{1}{2}\vec{q}_0)(\vec{\sigma}(3) \cdot \frac{1}{2}\vec{q}_0) \\ & - i[(\vec{\sigma}(2) \cdot \vec{q}_2)(\vec{\sigma}(3) \times \vec{\sigma}(1)) \cdot \vec{q}_2] + (\vec{\sigma}(2) \cdot \vec{q}_2)(\vec{\sigma}(3) \times \vec{\sigma}(1)) \cdot \frac{1}{2}\vec{q}_0 \\ & + (\vec{\sigma}(2) \cdot \frac{1}{2}\vec{q}_0)(\vec{\sigma}(3) \times \vec{\sigma}(1)) \cdot \vec{q}_2 + (\vec{\sigma}(2) \cdot \frac{1}{2}\vec{q}_0)(\vec{\sigma}(3) \times \vec{\sigma}(1)) \cdot \frac{1}{2}\vec{q}_0 \\ & - \frac{1}{3}(\vec{q}_2 + \frac{1}{2}\vec{q}_0)^2(\vec{\sigma}(3) \cdot \vec{\sigma}(1)) \end{aligned} \quad (B4)$$

$$C_{211} = \vec{\sigma}(2) \cdot \vec{\sigma}(3) \quad (B5)$$

$$\begin{aligned} C_{212} = & -\frac{1}{3}(\vec{q}_2 + \frac{1}{2}\vec{q}_0)^2(\vec{\sigma}(2) \cdot \vec{\sigma}(3)) \\ & + (\vec{\sigma}(1) \cdot \vec{q}_2)(\vec{\sigma}(3) \cdot \vec{q}_2) + (\vec{\sigma}(1) \cdot \vec{q}_2)(\vec{\sigma}(3) \cdot \frac{1}{2}\vec{q}_0) \\ & + (\vec{\sigma}(1) \cdot \frac{1}{2}\vec{q}_0)(\vec{\sigma}(3) \cdot \vec{q}_2) + (\vec{\sigma}(1) \cdot \frac{1}{2}\vec{q}_0)(\vec{\sigma}(3) \cdot \frac{1}{2}\vec{q}_0) \\ & - i[(\vec{\sigma}(1) \cdot \vec{q}_2)(\vec{\sigma}(3) \times \vec{\sigma}(2)) \cdot \vec{q}_2] + (\vec{\sigma}(1) \cdot \vec{q}_2)(\vec{\sigma}(3) \times \vec{\sigma}(2)) \cdot \frac{1}{2}\vec{q}_0 \\ & + (\vec{\sigma}(1) \cdot \frac{1}{2}\vec{q}_0)(\vec{\sigma}(3) \times \vec{\sigma}(2)) \cdot \vec{q}_2 + (\vec{\sigma}(1) \cdot \frac{1}{2}\vec{q}_0)(\vec{\sigma}(3) \times \vec{\sigma}(2)) \cdot \frac{1}{2}\vec{q}_0 \end{aligned} \quad (B6)$$

$$C_{221} = \vec{\sigma}(1) \cdot \vec{\sigma}(2) - i\vec{\sigma}(3) \cdot \vec{\sigma}(1) \times \vec{\sigma}(2) \quad (B7)$$

$$\begin{aligned} C_{222} = & -i(\vec{\sigma}(3) \cdot (\vec{q}_2 + \frac{1}{2}\vec{q}_0)(\vec{\sigma}(2) \times \vec{\sigma}(1)) \cdot (\vec{q}_2 + \frac{1}{2}\vec{q}_0) \\ & - \frac{1}{3}(\vec{q}_2 + \frac{1}{2}\vec{q}_0)^2[(\vec{\sigma}(1) \cdot \vec{\sigma}(2)) - i\vec{\sigma}(3) \cdot (\vec{\sigma}(1) \times \vec{\sigma}(2))] \\ & + (\vec{q}_2 + \frac{1}{2}\vec{q}_0)^2[1 + \vec{\sigma}(1) \cdot \vec{\sigma}(3) - \vec{\sigma}(1) \cdot \vec{\sigma}(2) - \vec{\sigma}(2) \cdot \vec{\sigma}(3)] \\ & + (\vec{\sigma}(1) \cdot (\vec{q}_2 + \frac{1}{2}\vec{q}_0))(\vec{\sigma}(2) \cdot (\vec{q}_2 + \frac{1}{2}\vec{q}_0)) - (\vec{\sigma}(1) \cdot (\vec{q}_2 + \frac{1}{2}\vec{q}_0))(\vec{\sigma}(3) \cdot (\vec{q}_2 + \frac{1}{2}\vec{q}_0)) \\ & + (\vec{\sigma}(2) \cdot (\vec{q}_2 + \frac{1}{2}\vec{q}_0))(\vec{\sigma}(3) \cdot (\vec{q}_2 + \frac{1}{2}\vec{q}_0)) \end{aligned} \quad (B8)$$

## APPENDIX C: THE ANGULAR DEPENDENCE OF Y

In order to see the angular dependence of a term like that given in (41) we use

$$\vec{\pi}(\vec{q}\vec{q}_2) = \frac{1}{2}\vec{q} + \vec{q}_2 \quad (\text{C1})$$

$$\vec{\pi}'(\vec{q}_2\vec{q}_0) = -\vec{q}_2 - \frac{1}{2}\vec{q}_0 \quad (\text{C2})$$

$$\vec{\pi}(\vec{q}_2\vec{q}_0) = \frac{1}{2}\vec{q}_2 + \vec{q}_0 \quad (\text{C3})$$

$$\vec{\pi}'(\vec{q}\vec{q}_2) = -\vec{q} - \frac{1}{2}\vec{q}_2 \quad (\text{C4})$$

Therefore

$$\hat{p} \cdot \hat{\pi}(\vec{q}\vec{q}_2) = \frac{\frac{1}{2}\hat{p} \cdot \vec{q} + \hat{p} \cdot \vec{q}_2}{|\frac{1}{2}\vec{q} + \vec{q}_2|} \quad (\text{C5})$$

$$|\pi'(\vec{q}\vec{q}_2)| = \sqrt{q_2^2 + \frac{1}{4}q_0^2 + \vec{q}_2 \cdot \vec{q}_0} \quad (\text{C6})$$

$$\hat{\pi}'(\vec{q}\vec{q}_2) \cdot \hat{\pi}(\vec{q}_2\vec{q}_0) = -\frac{\frac{1}{2}\vec{q} \cdot \vec{q}_2 + \vec{q} \cdot \vec{q}_0 + \frac{1}{4}q_2^2 + \frac{1}{2}\vec{q}_2 \cdot \vec{q}_0}{|\vec{q} + \frac{1}{2}\vec{q}_2||\frac{1}{2}\vec{q}_2 + \vec{q}_0|} \quad (\text{C7})$$

We put  $\hat{q} = \hat{z}$  and define

$$x = \hat{q}_2 \cdot \hat{q} \quad (\text{C8})$$

$$x_p = \hat{p} \cdot \hat{q} \quad (\text{C9})$$

$$\hat{p} \cdot \hat{q}_2 = x_p x + \sqrt{1 - x_p^2} \sqrt{1 - x^2} \cos(\phi_p - \phi_2) \quad (\text{C10})$$

$$x_{q_0} = \hat{q}_0 \cdot \hat{q} \quad (\text{C11})$$

$$\hat{q}_2 \cdot \hat{q}_0 = x x_{q_0} + \sqrt{1 - x^2} \sqrt{1 - x_{q_0}^2} \cos(\phi_2 - \phi_{q_0}) \quad (\text{C12})$$

This settles the angular dependencies of Y.

## APPENDIX D: TREATMENT OF SINGULARITIES

Regarding the expression (57) we face two types of integrals, where  $G_0$  appears alone or together with the deuteron pole:

$$H_1 \equiv \int d^3q_2 g(\vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), \vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2\vec{q}_0), \vec{\pi}'(\vec{q}_2\vec{q}_0)) G_0(\vec{\pi}'(\vec{q}\vec{q}_2)\vec{q}_2) \quad (\text{D1})$$

$$H_2 \equiv \int d^3q_2 h(\vec{p}, \vec{\pi}(\vec{q}\vec{q}_2), \vec{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2\vec{q}_0), \vec{\pi}'(\vec{q}_2\vec{q}_0)) \frac{1}{G_0(\vec{\pi}'(\vec{q}\vec{q}_2)\vec{q}_2) E_{q_2} + i\epsilon - E_d} \quad (\text{D2})$$

Here  $g$  and  $h$  are regular scalar functions. In order to arrive at the new way [21] one has to go back and rewrite both integrals into

$$H_1 \equiv \int d^3 q_2 \int d^3 p' \int d^3 p_2 \delta(\vec{p}' - \vec{\pi}(\vec{q}\vec{q}_2)) \delta(\vec{p}_2 - \vec{\pi}'(\vec{q}\vec{q}_2)) g(\vec{p}, \vec{p}', \vec{p}_2, \vec{\pi}(\vec{q}_2 \vec{q}_0), \vec{\pi}'(\vec{q}_2 \vec{q}_0)) G_0(\vec{p}_2, q_2) \quad (D3)$$

$$H_2 \equiv \int d^3 q_2 \int d^3 p' \int d^3 p_2 \delta(\vec{p}' - \vec{\pi}(\vec{q}\vec{q}_2)) \delta(\vec{p}_2 - \vec{\pi}'(\vec{q}\vec{q}_2)) h(\vec{p}, \vec{p}', \vec{p}_2, \vec{\pi}(\vec{q}_2 \vec{q}_0), \vec{\pi}'(\vec{q}_2 \vec{q}_0)) G_0(\vec{p}_2, \vec{q}_2) \frac{1}{E_{q_2} + i\epsilon - E_d} \quad (D4)$$

Then we change the product of the following two  $\delta$ -functions

$$\begin{aligned} & \delta(p' - \pi(\vec{q}\vec{q}_2)) \delta(p_2 - \pi'(\vec{q}\vec{q}_2)) \\ &= \delta(p' - \sqrt{\frac{1}{4}q^2 + q_2^2 + qq_2x}) \delta(p_2 - \sqrt{q^2 + \frac{1}{4}q_2^2 + qq_2x}) \\ &= \frac{2p'}{qq_2} \delta(x - x_0) \Theta(1 - |x_0|) \\ & \quad \delta(p_2 - \sqrt{\frac{3}{4}q^2 - \frac{3}{4}q_2^2 + p'^2}) \Theta(\frac{3}{4}q^2 - \frac{3}{4}q_2^2 + p'^2) , \end{aligned} \quad (D5)$$

with

$$x_0 = \frac{p'^2 - \frac{1}{4}q^2 - q_2^2}{qq_2} . \quad (D6)$$

We start with  $H_1$  and insert (D5)

$$\begin{aligned} H_1 &= \int d^3 q_2 \int d\hat{p}' \int d\hat{p}_2 \int dp' dp_2 \frac{2p'}{qq_2} \delta(x - x_0) \Theta(1 - |x_0|) \\ & \quad \delta(p_2 - \sqrt{\frac{3}{4}q^2 - \frac{3}{4}q_2^2 + p'^2}) \Theta(\frac{3}{4}q^2 - \frac{3}{4}q_2^2 + p'^2) g(\vec{p}, \vec{p}', \vec{p}_2, \vec{\pi}(\vec{q}_2 \vec{q}_0), \vec{\pi}'(\vec{q}_2 \vec{q}_0)) \\ & \quad \frac{1}{E + i\epsilon - \frac{3}{4m}q_2^2 - \frac{p_2^2}{m}} \delta(\hat{p}' - \hat{\pi}(\vec{q}\vec{q}_2)) \delta(\hat{p}_2 - \hat{\pi}'(\vec{q}\vec{q}_2)) . \end{aligned} \quad (D7)$$

Then we carry out the  $p_2, \hat{p}_2$  and  $\hat{p}'$  integrations

$$\begin{aligned} H_1 &= \frac{2}{q} \int d\hat{q}_2 \int dp' p' dq_2 q_2 \delta(x - x_0) \Theta(1 - |x_0|) \\ & \quad \Theta(\frac{3}{4}q^2 - \frac{3}{4}q_2^2 + p'^2) g(\vec{p}, p' \hat{\pi}(\vec{q}\vec{q}_2), p_2 \hat{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2 \vec{q}_0), \vec{\pi}'(\vec{q}_2 \vec{q}_0)) \\ & \quad \frac{1}{E + i\epsilon - \frac{3}{4m}q_2^2 - \frac{1}{m}(\frac{3}{4}q^2 - \frac{3}{4}q_2^2 + p'^2)} \\ &= \frac{2}{q} \int dp' p' \int dq_2 q_2 \Theta(1 - |x_0|) \Theta(\frac{3}{4}q^2 - \frac{3}{4}q_2^2 + p'^2) \\ & \quad \tilde{g}(p, p', q, q_2) \frac{1}{E + i\epsilon - \frac{3}{4m}q^2 - \frac{1}{m}p'^2} , \end{aligned} \quad (D8)$$

with

$$p_2 = \sqrt{\frac{3}{4}(q^2 - q_2^2) + p'^2} , \quad (D9)$$

and

$$\begin{aligned} & \tilde{g}(p, p', q, q_2) \\ &= \int d\hat{q}_2 \delta(x - x_0) g(\vec{p}, p' \hat{\pi}(\vec{q}\vec{q}_2), p_2 \hat{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2 \vec{q}_0), \vec{\pi}'(\vec{q}_2 \vec{q}_0)) . \end{aligned} \quad (\text{D10})$$

Remember  $x = \hat{q}_2 \cdot \hat{q}$ , thus it appears natural to choose  $\hat{q} = \hat{z}$  and then the  $x$ -integration can be carried out trivially.

The two  $\Theta$ -functions restrict the integrations in  $q_2$  and  $p'$  into an area whose size depends on the magnitudes of the spectator momentum  $q$ . It results

$$\begin{aligned} H_1 &= \frac{2}{q} \int_0^\infty dp' p' \frac{1}{E + i\epsilon - \frac{3}{4m} q^2 - \frac{1}{m} p'^2} \\ & \int_{|\frac{q}{2} - p'|}^{\frac{q}{2} + p'} dq_2 q_2 \tilde{g}(p, p', q, q_2) . \end{aligned} \quad (\text{D11})$$

The important point is that the free propagator appears now as a simple pole.

Next comes the  $H_2$ -integral, where two singular denominators appear. We rewrite it using again (D5) and obtain

$$\begin{aligned} H_2 &= \int d^3 q_2 \int d\hat{p}' \int d\hat{p}_2 \int dp' dp_2 \frac{2p'}{qq_2} \delta(x - x_0) \Theta(1 - |x_0|) \\ & \delta(p_2 - \sqrt{\frac{3}{4}q^2 - \frac{3}{4}q_2^2 + p'^2}) \Theta(\frac{3}{4}q^2 - \frac{3}{4}q_2^2 + p'^2) h(\vec{p}, \vec{p}', \vec{p}_2, \vec{\pi}(\vec{q}_2 \vec{q}_0), \vec{\pi}'(\vec{q}_2 \vec{q}_0)) \\ & \frac{1}{E + i\epsilon - \frac{3}{4m} q^2 - \frac{p_2^2}{m}} \frac{1}{E_{q_2} + i\epsilon - E_d} \delta(\hat{p}' - \hat{\pi}(\vec{q}\vec{q}_2)) \delta(\hat{p}_2 - \hat{\pi}'(\vec{q}\vec{q}_2)) . \end{aligned} \quad (\text{D12})$$

Then we carry out again the  $p_2, \hat{p}_2$  and  $\hat{p}'$  integrations

$$\begin{aligned} H_2 &= \frac{2}{q} \int d\hat{q}_2 \int dp' p' dq_2 q_2 \delta(x - x_0) \Theta(1 - |x_0|) \\ & \Theta(\frac{3}{4}q^2 - \frac{3}{4}q_2^2 + p'^2) \tilde{h}(p, p', q, q_2) \\ & \frac{1}{E + i\epsilon - \frac{3}{4m} q^2 - \frac{p'^2}{m}} \frac{1}{E_{q_2} + i\epsilon - E_d} , \end{aligned} \quad (\text{D13})$$

with

$$\begin{aligned} & \tilde{h}(p, p', q, q_2) \\ &= \int d\hat{q}_2 \delta(x - x_0) h(\vec{p}, p' \hat{\pi}(\vec{q}\vec{q}_2), p_2 \hat{\pi}'(\vec{q}\vec{q}_2), \vec{\pi}(\vec{q}_2 \vec{q}_0), \vec{\pi}'(\vec{q}_2 \vec{q}_0)) . \end{aligned} \quad (\text{D14})$$

We rewrite

$$\begin{aligned} & \frac{1}{E + i\epsilon - \frac{3}{4m} q^2 - \frac{p'^2}{m}} \frac{1}{E + i\epsilon - \frac{3}{4m} q_2^2 - E_d} \\ &= \left[ \frac{1}{E + i\epsilon - \frac{3}{4m} q^2 - \frac{p'^2}{m}} - \frac{1}{E + i\epsilon - \frac{3}{4m} q_2^2 - E_d} \right] \end{aligned}$$

$$\frac{1}{-E_d - \frac{3}{4m}q_2^2 + \frac{1}{m}(p'^2 + \frac{3}{4}q^2)} . \quad (\text{D15})$$

The new denominator function

$$\tilde{G}(q, q_2, p') \equiv \frac{1}{-E_d - \frac{3}{4m}q_2^2 + \frac{1}{m}(p'^2 + \frac{3}{4}q^2)} \quad (\text{D16})$$

cannot become singular inside the integration domain  $p' - q_2$ . Using the expression (D9) we see that

$$\tilde{G}(q, q_2, p') = \frac{1}{-E_d + \frac{1}{m}p_2^2} = \frac{1}{|E_d| + \frac{1}{m}p_2^2} . \quad (\text{D17})$$

Therefore (D13) turns into

$$\begin{aligned} H_2 = & \frac{2}{q} \int d\hat{q}_2 \int dp' p' dq_2 q_2 \delta(x - x_0) \Theta(1 - |x_0|) \\ & \Theta(\frac{3}{4}q^2 - \frac{3}{4}q_2^2 + p'^2) \tilde{h}(p, p', q, q_2) \\ & \left[ \frac{1}{E + i\epsilon - \frac{3}{4m}q^2 - \frac{p'^2}{m}} - \frac{1}{E + i\epsilon - \frac{3}{4m}q_2^2 - E_d} \right] \tilde{G}(q, q_2, p') \end{aligned} \quad (\text{D18})$$

We use the integration over the area in the  $p' - q_2$  plane and obtain

$$\begin{aligned} H_2 = & \frac{2}{q} \int_0^\infty dp' p' \frac{1}{E + i\epsilon - \frac{3}{4m}q^2 - \frac{p'^2}{m}} \tilde{h}(p, p', q, q_2) \int_{|\frac{q}{2} - p'|}^{\frac{q}{2} + p'} dq_2 q_2 \tilde{G}(q, q_2, p') \\ & - \frac{2}{q} \int_0^\infty dq_2 q_2 \frac{1}{E + i\epsilon - \frac{3}{4m}q_2^2 - E_d} \tilde{h}(p, p', q, q_2) \int_{|\frac{q}{2} - q_2|}^{\frac{q}{2} + q_2} dp' p' \tilde{G}(q, q_2, p') . \end{aligned} \quad (\text{D19})$$

In the second integral we integrated first over  $q_2$  and then over  $p'$ . In both cases we have just a simple pole either in  $p'$  or in  $q_2$ .

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- [1] W. Glöckle, H. Witała, D.Hüber, H. Kamada, J. Golak, Phys. Rep. **274**, 107 (1996).
  - [2] A. Kievsky, M. Viviani, S. Rosati, D. Hüber, W. Glöckle, H. Kamada, H. Witała, J. Golak, Phys. Rev. **C58**, 3085 (1998).
  - [3] H. Witała, W. Glöckle, J. Golak, A. Nogga, H. Kamada, R. Skibiński, and J. Kuroś-Żołnierczuk, Phys. Rev. **C 63**, 024007 (2001); J. Kuroś-Żołnierczuk, H. Witała, J. Golak, H. Kamada, A. Nogga, R. Skibiński, W. Glöckle, Phys. Rev. **C 66**, 024004 (2002), and references therein.
  - [4] K. Sekiguchi et al., Phys. Rev. **C 79**, 054008 (2009); and references therein.
  - [5] E. Stephan et al., Phys. Rev. **C 76**, 057001 (2007); and references therein.
  - [6] R.B. Wiringa, V.G.J. Stoks, R. Schiavilla, Phys. Rev. **C51**, 38 (1995).
  - [7] R. Machleidt, Phys. Rev. **C63**, 024001 (2001).
  - [8] V.G.J. Stoks et al., Phys. Rev. **C49**, 2950 (1994).



- [9] S. A. Coon, H. K. Han, Few-Body Syst. **30**, 131 ( 2001); B. S. Pudliner, V. R. Pandariphande, J. Carlson, Steven C. Pieper, R.B. Wiringa, Phys. Rev.**C56**, 1720 (1997).
- [10] E. Epelbaum, Prog. Part. Nucl. Phys. **57**, 654 (2006).
- [11] E. Epelbaum, A. Nogga, W. Glöckle, H. Kamada, Ulf-G. Meißner, H. Witała, Phys. Rev. **C66**, 064001 (2002); E. Epelbaum, A. Nogga, H. Witała, H. Kamada, W. Glöckle, Ulf-G. Meißner, Eur. Phys. J.A. **17**, 415 (2003).
- [12] A. Nogga et al., Nucl. Phys. **A 737**, 236 (2004); P. Navratil et al., Phys. Rev. Lett. **99**, 042501 (2007).
- [13] H. Liu, C. Elster and W. Glöckle, Phys. Rev. C **72**, 054003 (2005)
- [14] T. Lin, C. Elster, W. N. Polyzou, H. Witała and W. Glöckle, Phys. Rev. C **78**, 024002 (2008)
- [15] T. Lin, C. Elster, W. N. Polyzou and W. Glöckle, Phys. Lett. B **660**, 345 (2008).
- [16] H. Witała, J. Golak, W. Glöckle, H. Kamada, Phys. Rev. **C 71**, 054001 (2005); H.Witała, J. Golak, and R. Skibiński, Phys. Lett. **B 634**, 374 (2006); R. Skibiński, H. Witała, J. Golak, Eur. Phys. J. **A 30**, 369 (2006).
- [17] H. Witała, J. Golak, R. Skibiński, W. Glöckle, W.N. Polyzou, H. Kamada, Phys. Rev. **C 77**, 034004 (2008).
- [18] W. Glöckle, Ch. Elster, J. Golak, R. Skibiński, H. Witała, H. Kamada, arXiv:0906.0321 (accepted for publication in Few Body Systems).
- [19] D.Hüber, H.Kamada, H.Witała, W.Glöckle, Acta Phys. Polonica **B28**, 1677 (1997).
- [20] H. Witała, W. Glöckle, Eur. Phys. J. **A37**, 87 (2008).
- [21] Ch. Elster, W. Glöckle, H. Witała, Few Body Syst. **45**, 1, (2009).
- [22] W. Glöckle, The Quantum Mechanical Few-Body Problem, Springer-Verlag, Berlin-Heidelberg, (1983).
- [23] C. Elster, T. Lin, W. Glöckle and S. Jeschonnek, Phys. Rev. C **78**, 034002 (2008).
- [24] H. Witała, R. Skibiński, J. Golak, W. Glöckle, Eur. Phys. J. **A41**, 369 (2009).
- [25] H. Witała, W. Glöckle, H. Kamada, Phys. Rev. **C43**, 1619 (1991).
- [26] I. Fachrudin et al., to be published.
- [27] L.Wolfenstein, Phys.Rev **96**, 1654 ( 1954); M. H. McGregor, M. J. Morawcsik, H. P. Stapp, Annu. Rev. Nucl. Sci. 10, 291 ( 1960).
- [28] I. Fachruddin, Ch. Elster, W. Glöckle, Phys. Rev. **C63**, 054003 (2001).